2103000206020034
EXAMINATION FEBRUARY-MARCH 2024
BACHELOR OF SCIENCE (SIXTH SEMESTER)
MATHEMATICS - IX (MTH - 604 - REAL ANALYSIS - IV)
LEVEL 2

[Time: As Per Schedule] **[Max. Marks: 50]**

Instructions:
1. Fill up strictly the following details on your answer book
  a. Name of the Examination : **BACHELOR OF SCIENCE (SIXTH SEMESTER)**
  b. Name of the Subject : **MATHEMATICS - IX (MTH – 604 - REAL ANALYSIS - IV) LEVEL 2**
  c. Subject Code No : **2103000206020034**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

Student's Signature

Q.1 Answer any FIVE as directed. **10**

(1) Is a Bounded subset of a metric space Totally bounded? Justify your answer.

(2) State Picard's fixed point Theorem.

(3) Consider [2,3] in R^1 with absolute value metric is [2,3] compact? Justify your answer.

(4) Show that infinite subset of R_d is not compact.

(5) Show that metric spaces $[0,1]$ and $[0,2]$ are Homeomorphic.

(6) Is $\left[\frac{1}{2},1\right)$ open in $[0,1]$? Justify your answer.

(7) If E is a subset of a metric space M, then give an example to show that the limit point of the set E need not be member of E.

(8) State Generalized Nested Interval Theorem.

Q.2 Attempt any TWO. 10

(1) Let $\langle M_1, \rho_1 \rangle$ and $\langle M_2, \rho_2 \rangle$ be metric spaces. Show that if f is continuous on M_1, then $f^{-1}(F)$ is closed subset of M_1 whenever F is closed subset of M_2.

(2) Show that the finite union of closed sets in any metric space closed. Is infinite union of closed sets closed? Justify your answer.

(3) Let A and B are subsets of a metric space M. Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$. Also prove that $\overline{A \cap B} \subset \bar{A} \cap \overline{\mathrm{B}}$.

Q.3 Attempt any TWO. 10

(1) Prove that a subset a of R_d is Totally bounded if and only if A contains only a finite number of points.

(2) Define characteristic function. Let M be a metric space. If every continuous characteristic function on M is constant, then show that M is connected.

(3) If A is a connected subset of a metric space M, and if $A \subset B \subset \bar{A}$, then prove that B is connected.

Q.4 Attempt any TWO. 10

(1) If $\langle M, \rho \rangle$ is a complete metric space and A is closed subset of M, then show that $\langle A, \rho \rangle$ is complete.

(2) If T: [0,1] $\rightarrow [0, 1]$ and there is a real $\propto$ with $0 \leq \propto \leq 1$ with such that $|T'(x)| \leq \propto$ $(0 \leq x \leq 1)$, where T' is derivative of T, then prove that T is a contraction map on $[0, 1]$.

(3) Define complete metric space. Show that any finite set of a complete metric space is complete.

Q.5 Attempt any TWO. **10**

(1) Let A be the subset of a metric space $\langle M, \rho \rangle$. If $\langle A, \rho \rangle$ is compact, then show that A is closed subset of $\langle M, \rho \rangle$.

(2) Prove that every finite subset of any metric space is compact.

(3) If the metric space M has the Heine-Borel property, then show that M is compact.

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